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Effects of transverse convection and solid–liquid density difference on the steady close-contact melting

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Abstract

The steady close-contact melting phenomenon occurring between a phase change material and an isothermally heated flat surface in relative motion is investigated analytically, with the effects of transverse convection across the liquid film and solid–liquid density difference taken into account. Scale analysis is used to estimate the dependence of system variables on characteristic parameters. Also, an analytical solution to a set of simplified model equations is obtained to quantify the effects. Transverse convection can be characterized by a dimensionless interfacial temperature gradient which asymptotically approaches unity/zero with decreasing/increasing the Stefan number. The convection effect in the liquid film can be neglected approximately for the range of the Stefan number less than 0.1. It is found that the solid descending velocity depends linearly on the liquid-to-solid density ratio, and that the ratios of solid descending velocity, film thickness and friction coefficient to the conduction solution are proportional to 3/4, 1/4 and −1/4 powers of the interfacial temperature gradient, respectively. © 1998 Published by Elsevier Science Inc. All rights reserved.

Keywords: Close-contact melting; Transverse convection; Solid–liquid density difference

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Notation

c	specific heat
f	friction coefficient, F_t/F_n
F_n	normal force, Fig. 1
$\tilde{F}_n$	dimensionless normal force, $F_nL/(\mu\alpha)$
F_t	tangential force, Fig. 1
h_{sf}	latent heat of fusion
k	thermal conductivity
L	sliding-contact length
P	pressure
ΔP	longitudinal pressure difference
Ste	Stefan number, $c\Delta T/h_{sf}$
T	temperature
T_m	melting point of phase change material
ΔT	transverse temperature difference
ΔT_x	longitudinal temperature difference
u, v	velocity components, Fig. 1
U	velocity of relative motion
$\tilde{U}$	dimensionless velocity of relative motion, UL/α
V	solid descending velocity
$\tilde{V}$	dimensionless solid descending velocity, VL/α
x, y	Cartesian coordinates, Fig. 1
z	variable standing for $\tilde{\rho}^{-1}\tilde{V}\tilde{\delta}$

Greek

α	thermal diffusivity, $k/(\rho_l c)$
δ	thickness of the liquid film
$\tilde{\delta}$	dimensionless thickness of the liquid film, δ/L
ζ	dummy variable
η	dimensionless vertical position, y/δ
μ	viscosity
ρ	density
$\tilde{\rho}$	liquid-to-solid density ratio, ρ_l/ρ_s
θ	dimensionless temperature, $(T - T_m)/\Delta T$
ϕ	dimensionless interfacial temperature gradient, Eq. (28)

Subscripts

cond	conduction solution
l	liquid phase
s	solid phase

1. Introduction

Contact melting is related to diverse applications such as latent heat storage using encapsulated phase change materials, melt lubrication by the surface coating of a metallic part with a low melting point material, interior ballistics associated with superficial melting of a projectile traveling along a gun barrel, and burial of heat-generating bodies.

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Recently, Bejan (1994) presented a comprehensive review on contact melting. Of special interest to the present study is the analytical modeling of close-contact melting occurring between a phase change material and a heated surface. In most of the previous modelings, it has been assumed that conduction normal to the surface, i.e. transverse (or crosswise) conduction, is the only heat transport mechanism in the liquid film formed between the two solids. However, the assumption seems to be valid only when the Stefan number is sufficiently small. This argument can be substantiated by the work of Hong and Saito (1993), which numerically simulated the transient behavior from the beginning to the steady-state of close-contact melting on an isothermal flat surface with a sophisticated model accounting for full convection. At the steady-state, both the film thickness and the solid descending velocity for $\mathrm{Ste} = 0.01266$ agree well with the existing conduction solution, whereas those for $\mathrm{Ste} = 1.266$ considerably underpredict the conduction solution. The discrepancy for the latter case was merely attributed to the effect of convection in the liquid film, but a definite verification has not been presented since then.

The solid–liquid density difference of phase change material in close-contact melting has been recognized as a driving force to keep contact between melting and heated solids by gravity in the absence of external force, e.g. Bareiss and Beer (1984). On the other hand, its effect on fluid mechanics in the liquid film has been inattentively treated in spite of the physical reality. Since the liquid flowing through the gap between two solids is actually generated by melting at the solid–liquid interface, the melting rate as well as the density difference must affect the volume flow rate and pressure in the film. In this regard, the role played by the density difference in close-contact melting needs to be assessed.

The present study is intended to clarify the effects of convection in the liquid film and solid–liquid density difference of phase change material in a representative steady close-contact melting process. In association with melt lubrication, the frictional characteristics resulting from relative (tangential) motion between two solid parts are also included in the model. First, scale analysis is performed not only to assure the simplification introduced in the modeling procedure, but also to derive qualitative relations between the system variables and relevant parameters. Then, an analytical solution to the simplified model equations is pursued to quantify the results. The effects of density difference and transverse convection are to be represented in terms of the liquid-to-solid density ratio and a dimensionless interfacial temperature gradient, respectively. In particular, the specific effects of transverse convection on the system variables are highlighted by examining the behavior of the interfacial temperature gradient and by comparing the present results with the conduction solution.

2. Model description

The physical system considered in this work, as depicted schematically in Fig. 1, is a simple one among various geometric configurations of contact melting. This system seems to be convenient for explaining the principles of close-contact melting and lubrication (Bejan, 1989, 1995) in the presence of convection and solid–liquid density difference, and can be readily extended to even more general cases (Bejan, 1992).

A block of solid-state phase change material at its melting point T_m melts quasi-steadily on the flat surface which is heated isothermally at a prescribed temperature $T_m + \Delta T$. Contact between two solids over the length L is maintained by the externally applied force F_n. During the contact melting process, the liquid melt created at a constant rate along the solid–liquid interface fills the gap, flows toward the ends, and eventually is squeezed out through the end openings. Since the liquid film thickness is invariant at the steady-state, the vertical (descending) velocity of solid block must coincide with the melting rate at the interface. Both the liquid film thickness δ and the solid descending velocity V are the target unknowns to be determined. In addition, when one of the solid parts moves at a known velocity U relative to the other, the tangential force F_t supporting the stationary part is another important variable pertaining to lubrication. Note that the present coordinate system fixed at one end of the stationary part (see Fig. 1) can also be applied to analyze the case that both parts are in motion by simply adjusting the relative velocity between them.

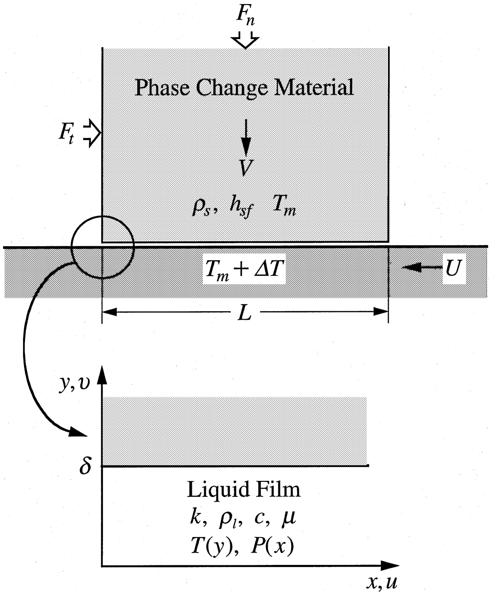

Fig. 1. Schematic of the contact-melting system considered in the present study.

A brief description of the fundamental features involved in contact melting process is useful for understanding the global picture of the present problem. Inside the thin liquid film, the vertically downward flow developed by the simultaneous actions of melting and solid descending motion has the maximum velocity at the melting front, gradually diminishes as it travels across the film, and finally vanishes on the heated surface on account of flow turning toward the openings. Crossing the gap, the temperature changes from the melting point to the surface temperature. At low melting rates, the linear temperature profile across the film corresponding to the conduction solution is nearly unaffected by crosswise convection. At higher rates, however, the melt ejected vigorously from the melting front cools down the temperature locally, reducing the interfacial temperature gradient. This statement is consistent with the previous work (Hong and Saito, 1993), in that the solid descending velocity predicted numerically is considerably smaller than that from the conduction solution. In regard of the effect of density difference, it has been reflected in the existing conduction solution in terms of the liquid-to-solid density ratio (Bejan, 1992, 1995). There still remains a question as to whether the density difference interacts with the transverse convection or not. No analytical model that simultaneously accounts for convection and density difference has been presented yet. It is important not only to quantify the effects of transverse convection and density difference, but also to establish the criterion to include either of them in the analysis.

In order to render the problem mathematically tractable, the following assumptions have been introduced:

(a) The process is quasi-steady, as already noted, so that at every moment of the process the normal force exerted on the solid block is balanced by the pressure of the liquid film. The unsteady behavior expected during the initial stage of contact melting (Hong and Saito, 1993) is percluded here.

(b) The film thickness is uniform in the direction tangent to the heated surface, which implies that the effect of longitudinal (horizontal) convection is negligible, even when that of transverse (vertical) convection is appreciable. The validity of this assumption will be discussed later.

(c) The velocity of relative motion U is so imposed that the effect of viscous dissipation (frictional heating) can be neglected. This assumption has been employed commonly in the previous models (Bejan, 1995), except the case that the viscous dissipation is the only heat source for melting when the velocity is high (Bejan, 1989).

3. Analysis

3.1. Model equations

Two of the system variables, i.e. the film thickness δ and the solid descending velocity V, can be determined simultaneously by the vertical force balance on the melting block,

$$F_\mathrm{n} = \int_0^L P(x)\,\mathrm{d}x \tag{1}$$

and the energy balance at the melting front,

$$-k\frac{\partial T}{\partial y}\bigg|_{y=\delta} = \rho_s h_{sf} V \tag{2}$$

The horizontal force balance relates the unknown tangential force F_t with the shear force acting on the heated surface,

$$F_\mathrm{t} = -\int_0^L \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}\mathrm{d}x \tag{3}$$

In Eqs. (1)–(3), the pressure distribution in the liquid film, the temperature gradient at the interface and the velocity gradient at the heated surface should be expressed in terms of the dependent variables (i.e. V, δ and F_t) for the closure of modeling. This can be done by solving the continuity, momentum and energy equations for the film. The continuity equation is expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{4}$$

According to the classical theory of lubrication (Bejan, 1989; Batchelor, 1967), the liquid inertia and the pressure variation in the transverse direction (y) are negligible, yielding a simplified momentum equation,

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \mu\frac{\partial^2 u}{\partial y^2}. \tag{5}$$

Relying on the assumption (c) and neglecting the longitudinal heat conduction, the energy equation reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{6}$$

Note that Eq. (6) still retains the convection terms which have been excluded in the conduction models (Bejan, 1989, 1994, 1995). The energy equation admits further simplification, which is described below.

3.2. Scale analysis

Order-of-magnitude of convection terms in the energy equation is estimated first. In the previous study (Bejan, 1995), the terms in Eq. (6) were scaled as $U\,\Delta T/L$, $V\,\Delta T/\delta$ and $\alpha\,\Delta T/\delta^2$. Scaling Eq. (4), i.e.

$$U/L \sim V/\delta, \tag{7}$$

yields the following order-of-magnitude relation from the energy equation,

$$U\,\Delta T/L \sim V\,\Delta T/\delta. \tag{8}$$

Eq. (8) indicates that the transverse and longitudinal convections are of the same order. On the basis of this relation and the assumption that the order of convection-to-conduction ratio is sufficiently small (i.e. $V\delta/\alpha \ll 1$), the convection terms were excluded in the conduction models.

The scaling relation, Eq. (8), does not pose any problem if the above assumption is valid. If not, both of the convection terms in Eq. (6) should be included in the analysis. Such a potentially complicated approach can be circumvented, however. The numerical simulation for $\mathrm{Ste} = 1.266$ (the relation, $V\delta/\alpha \sim \mathrm{Ste}$, will be identified later) in Hong and Saito (1993) shows that the film thickness remains nearly uniform in the horizontal direction (within 0.1% of relative variation), while deviating considerably from the conduction solution. This implies that only the crosswise convection affects the temperature distribution in the film, since the film thickness depends directly on the local temperature gradient at the melting front. Apart from the numerical evidence, it is physically plausible that the temperature difference in each direction is of distinct order. Considering that flow through the liquid film is laminar, and that both boundaries, i.e. the solid–liquid interface and the heated surface, are kept isothermal, the longitudinal temperature difference must be much smaller than the transverse counterpart. If this is the case, the scaling relation for convection terms, Eq. (8), should be replaced by

$$U\,\Delta T_x/L \ll V\,\Delta T/\delta, \tag{9}$$

where ΔT_x denotes a newly estimated longitudinal temperature difference. Relying on Eq. (9), Eq. (6) is further simplified as

$$v\frac{\mathrm{d}T}{\mathrm{d}y} = \alpha\frac{\mathrm{d}^2 T}{\mathrm{d}y^2} \tag{10}$$

which is valid even when $V\delta/\alpha \sim 1$. The foregoing discussion seems to suffice to validate the assumption (b) described earlier.

From now on, the focus is placed on the characteristics of contact melting in the simultaneous presence of crosswise convection and solid–liquid density difference. Scaling Eqs. (4) and (5) gives:

$$U/L \sim (\rho_s/\rho_l)V/\delta, \tag{11}$$

$$\Delta P/L \sim \mu U/\delta^2, \tag{12}$$

respectively. Note that the scale of vertical liquid velocity in Eq. (11) is taken as $(\rho_s/\rho_l)V$ instead of V in order to incorporate the effect of density difference. Substituting Eqs. (11) and (12) into Eq. (1), we have the following relation,

$$F_\mathrm{n} \sim L\Delta P \sim \mu(\rho_s/\rho_l)VL^3/\delta^3. \tag{13}$$

The order-of-magnitude relation corresponding to Eq. (2) can be expressed as

$$k\Delta T/\delta \cdot \phi \sim \rho_s h_{sf} V, \tag{14}$$

where the term ϕ, which is defined later, is a dimensionless temperature gradient at the interface. It physically represents the degree of change in the temperature gradient of pure conduction, $k\Delta T/\delta$, due to crosswise convection.

From Eqs. (13) and (14), with the aid of the definitions of dimensionless quantities, the dependent variables are finally scaled as

$$\tilde{V} \sim \tilde{\rho}(\mathrm{Ste}\cdot\phi)^{3/4}\tilde{F}_{\mathrm{n}}^{1/4}, \tag{15}$$

$$\tilde{\delta} \sim (\mathrm{Ste}\cdot\phi)^{1/4}\tilde{F}_{\mathrm{n}}^{-1/4}. \tag{16}$$

Three observations can be made from these results. First, the transverse convection affects both the solid descending velocity $\tilde{V}$ and the liquid film thickness $\tilde{\delta}$ in the consolidated form with the Stefan number, i.e. $\mathrm{Ste}\cdot\phi$. Second, the density ratio $\tilde{\rho}$ alters $\tilde{V}$ only, provided ϕ is not a function of $\tilde{\rho}$. Finally, Eqs. (15) and (16) reduce to the existing conduction solutions without density difference (Bejan, 1992; Bejan, 1995) when $\phi = 1$ and $\tilde{\rho} = 1$. Moreover, it can be asserted at this stage that an uncertainty regarding the density ratio found in Bejan (1992), specifically $\tilde{\rho}^{3/4}$ in place of $\tilde{\rho}$ in Eq. (15), originated from the use of Eq. (7) instead of Eq. (11) there.

3.3. Analytical solution

In order to obtain quantitative informations, an analytical solution is sought for a set of simplified model Eqs. (1)–(5) and (10). The fluid mechanics part of the analysis is quite straightforward, as in the previous studies (Bejan, 1989, 1995). Eq. (5) subject to no-slip boundary conditions,

$$u = 0 \text{ at } y = 0 \text{ and } \delta \tag{17}$$

is solved to yield the parabolic velocity profile in the film,

$$u(x,y) = \frac{\delta^2}{2\mu}\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)(\eta^2 - \eta) + U(1-\eta). \tag{18}$$

Integration of Eq. (4) across the film using Eq. (18) and the boundary conditions for v,

$$v = 0 \text{ at } y = 0 \quad \text{and} \quad v = -(\rho_s/\rho_l)V \text{ at } y = \delta, \tag{19}$$

gives

$$\frac{\delta^3}{12\mu}\left(-\frac{\mathrm{d}^2P}{\mathrm{d}x^2}\right) = \left(\frac{\rho_s}{\rho_l}\right)V. \tag{20}$$

It is easy to integrate Eq. (20) subject to the end conditions ($P = 0$ at $x = 0$ and L) with respect to x,

$$P(x) = 6\mu(\rho_s/\rho_l)(V/\delta^3)(Lx - x^2). \tag{21}$$

The known pressure distribution along the film facilitates the integration appeared in Eq. (1), which allows the force balance on the solid block to be expressed in dimensionless form as

$$\tilde{F}_{\mathrm{n}} = \tilde{\rho}^{-1}\tilde{V}\tilde{\delta}^{-3}. \tag{22}$$

The vertical velocity profile $v(y)$, which is prerequisite to deriving the temperature distribution across the film, is readily obtained. By substituting Eq. (18) into Eq. (4), we have a differential equation for $v(y)$,

$$\frac{\partial v}{\partial y} = -\frac{\delta^2}{2\mu}\left(\frac{\mathrm{d}^2P}{\mathrm{d}x^2}\right)(\eta^2 - \eta), \tag{23}$$

the solution of which is

$$v(y) = (\rho_s/\rho_l)V(2\eta^3 - 3\eta^2). \tag{24}$$

Note that the above profile satisfies the boundary conditions, Eq. (19). Thus the energy equation, Eq. (10), subject to the boundary conditions,

$$T = T_{\mathrm{m}} + \Delta T \text{ at } y = 0 \quad \text{and} \quad T = T_{\mathrm{m}} \text{ at } y = \delta \tag{25}$$

is ready to be solved. The solution, the temperature distribution across the film, can be written as

$$\theta = 1 - \int_0^{\eta} \mathrm{e}^{\tilde{\rho}^{-1}\tilde{V}\tilde{\delta}\left(\zeta^4/2-\zeta^3\right)}\,\mathrm{d}\zeta \Bigg/ \int_0^{1} \mathrm{e}^{\tilde{\rho}^{-1}\tilde{V}\tilde{\delta}\left(\zeta^4/2-\zeta^3\right)}\,\mathrm{d}\zeta, \tag{26}$$

where ζ is a dummy variable for integration.

Since the temperature gradient at the melting front can be obtained from Eq. (26), Eq. (2) simply reduces to

$$\tilde{\rho}^{-1}\tilde{V}\tilde{\delta} = \mathrm{Ste}\cdot\phi, \tag{27}$$

where the dimensionless interfacial temperature gradient ϕ is defined by

$$\phi = -\left.\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right|_{\eta=1} = \mathrm{e}^{-\tilde{\rho}^{-1}\tilde{V}\tilde{\delta}/2} \Bigg/ \int_0^{1} \mathrm{e}^{\tilde{\rho}^{-1}\tilde{V}\tilde{\delta}\left(\zeta^4/2-\zeta^3\right)}\,\mathrm{d}\zeta. \tag{28}$$

An apparent dependence of ϕ on the density ratio $\tilde{\rho}$ fades away by introducing a supplementary variable $z = \tilde{\rho}^{-1}\tilde{V}\tilde{\delta}$, and by rearranging Eq. (27) into

$$z\cdot\mathrm{e}^{z/2}\int_0^{1} \mathrm{e}^{z\left(\zeta^4/2-\zeta^3\right)}\,\mathrm{d}\zeta = \mathrm{Ste}. \tag{29}$$

Note that the variable z is a function of Ste only. If z is known for a prescribed Ste as a solution of Eq. (29), the term ϕ is determined from Eq. (27), i.e. $\phi = z/\mathrm{Ste}$. Eq. (29) is readily solved, e.g. via the Newton–Raphson method, since the LHS of it is a monotonically increasing function of z. In consequence, ϕ is also a function of Ste only. This fact indicates that the effect of transverse convection and density difference are mutually independent, being characterized by the interfacial temperature gradient and the density ratio, respectively, Finally, Eqs. (22) and (27) are solved to yield

$$\tilde{V} = \tilde{\rho}(\mathrm{Ste}\cdot\phi)^{3/4}\tilde{F}_{\mathrm{n}}^{1/4}, \tag{30}$$

$$\tilde{\delta} = (\mathrm{Ste}\cdot\phi)^{1/4}\tilde{F}_{\mathrm{n}}^{-1/4}. \tag{31}$$

These results are consistent with those of scale analysis, except that ϕ is specified here.

The tangential force is conveniently represented by the friction coefficient defined as $f = F_{\mathrm{t}}/F_{\mathrm{n}}$ (Bejan, 1989; Bejan, 1995). Applying the velocity profile, Eq. (18), to the horizontal force balance, Eq. (3), we have $F_{\mathrm{t}} = \mu UL/\delta$, thereby

$$f = \tilde{U}(\mathrm{Ste}\cdot\phi)^{-1/4}\,\tilde{F}_{\mathrm{n}}^{-3/4}. \tag{32}$$

The friction coefficient f, which depends on the crosswise convection only (regardless of the density difference), also reduces to the conduction solution (Bejan, 1989, 1995) when $\phi = 1$, in the same manner as $\tilde{V}$ and $\tilde{\delta}$.

4. Discussion

Recalling that the interfacial temperature gradient ϕ is a function of Ste only, the final results, Eqs. (30)–(32), show that only the density difference affects the solid descending velocity. That is, the velocity $\tilde{V}$ depends linearly on the ratio $\tilde{\rho}$. This can be interpreted physically as follows. Consider the contact melting process of a typical phase change material with $\tilde{\rho} < 1$, i.e. the solid phase is denser than the liquid phase. In such a situation, the excess liquid is generated along the solid–liquid interface compared with the case of $\tilde{\rho} = 1$, resulting in the rise of pressure in the film the thickness of which is kept constant. The pressure rise, in turn, causes the solid descending velocity to decrease to meet the force balance on the block for a constant normal force applied externally.

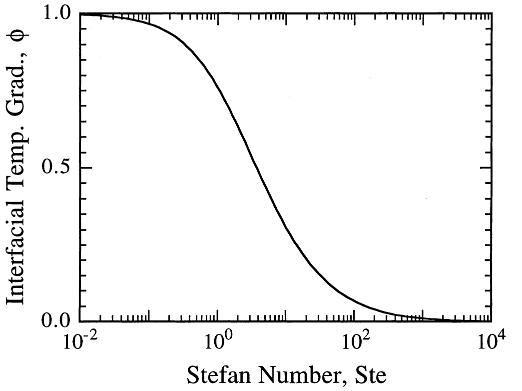

Fig. 2. Asymptotic behaviors of the dimensionless interfacial temperature gradient as a function of the Stefan number.

Since the quantitative effect of transverse convection is represented by ϕ, it is meaningful to investigate the behavior of ϕ(Ste) in advance. Physically, $\phi = 1$ corresponds to pure conduction, and ϕ is expected to decrease with intensifying the transverse convection. Fig. 2 depicts the calculated ϕ(Ste), in which the range of large Ste is included merely to illustrate the limiting behavior, although such conditions cannot be realized. It is no surprise that the term asymptotically approaches unity/zero with decreasing/increasing the Stefan number.

In order to visualize the variation of temperature gradient with respect to Ste, temperature profiles for a number of selected cases are shown Fig. 3. With increasing the Stefan number, the downward convective flow starting at the solid–liquid interface intensifies, so that the temperature profile across the film tends to deviate from the straight line corresponding to pure conduction, especially in the vicinity of the interface. A significant result of this work is to show the importance of convection.

Before discussing the effect of transverse convection, its scale is estimated. According to scale analysis, the ratio of transverse convection to conduction is of order of $(\rho_s/\rho_l)V\delta/\alpha$ in the presence of density difference, which is equal to the LHS of Eq. (27) in dimensionless form. In fact, the consolidated factor Ste $\cdot\,\phi$ denotes the scale of convection-to-conduction ratio. Note here that the aforementioned relation $V\delta/\alpha \sim$ Ste is valid owing to ϕ(Ste) even in the presence of density difference. Fig. 4 shows variation of the ratio Ste $\cdot\,\phi$ within the physically meaningful range of Ste. It is natural that the effect of transverse convection becomes significant with increasing Ste. Approximately for Ste > 1.1, the contribution of convection appears to predominate over that of conduction, which, however, does not necessarily reflect the physical reality. The convection-to-conduction ratio represented by the curve in Fig. 4 should be understood in an order-of-magnitude sense.

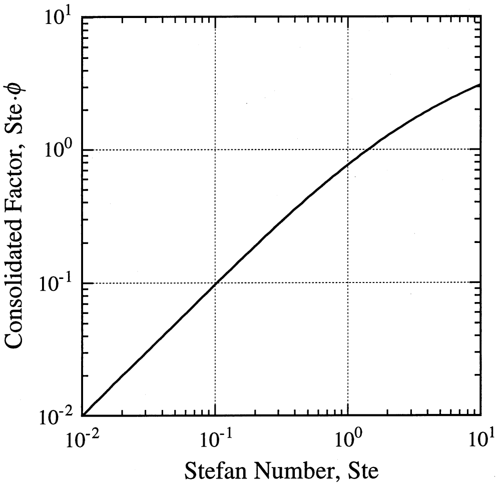

Fig. 4. Variation in the order-of-magnitude ratio of transverse convection to conduction with respect to the Stefan number.

It is convenient to express the quantitative effect of transverse convection by the ratio of the present solution, Eqs. (30)–(32), to the corresponding conduction solution ($\phi = 1$) under the same melting conditions. The ratio of each system variable is simply expressed as

$$V/V_{\text{cond}} = \phi^{3/4}, \tag{33}$$

$$\delta/\delta_{\text{cond}} = \phi^{1/4}, \tag{34}$$

$$f/f_{\text{cond}} = \phi^{-1/4}, \tag{35}$$

respectively. All of them are functions of the interfacial temperature gradient ϕ only, and are plotted in Fig. 5 for the same

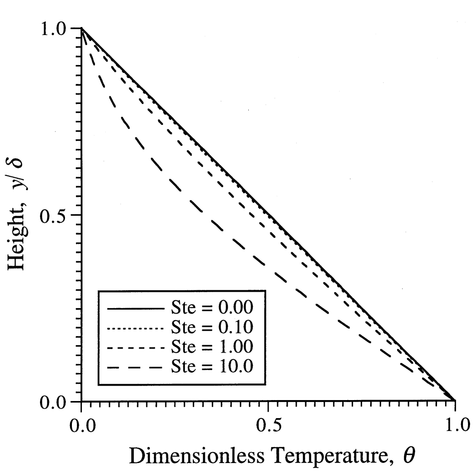

Fig. 3. Temperature profiles across the liquid film at different Stefan numbers.

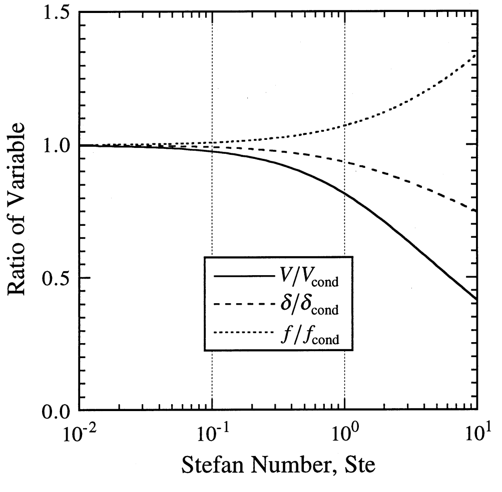

Fig. 5. Effect of transverse convection on the system variables as a function of the Stefan number.

range of Ste with Fig. 4. As the crosswise convection intensifies, both the solid descending velocity and the film thickness decrease, but the friction coefficient increases, in comparison with the conduction solution. Due to the asymptotic behavior of ϕ, the ratios keep nearly constant, i.e. unity, approximately for the range of $\mathrm{Ste} < 0.1$, where the conduction solution is valid within a tolerance. Outside the range, however, the ratios deviate appreciably from unity depending on the exponent. These results substantiate that the key reason for underpredictions in both the solid descending velocity and the film thickness compared with the pure conduction solution for $\mathrm{Ste} = 1.266$ in the previous work (Hong and Saito, 1993) is the effect of transverse convection.

References

Bareiss, M., Beer, H., 1984. An analytical solution of the heat transfer process during melting of an unfixed solid phase change material inside a horizontal tube. Int. J. Heat Mass Transfer 27, 739–746.

Batchelor, G.K., 1967. An Introduction to Fluid Dynamics. Cambridge University Press, Cambridge, UK, pp. 219–222.

Bejan, A., 1989. The fundamentals of sliding contact melting and friction. J. Heat Transfer 111, 13–20.

Bejan, A., 1992. Single correlation for theoretical contact melting results in various geometries. Int. Comm. Heat Mass Transfer 19, 472–483.

Bejan, A., 1994. Contact melting heat transfer and lubrication. Advances in Heat Transfer 24, 1–38.

Bejan, A., 1995. Convection Heat Transfer 2nd ed., Wiley, New York, pp. 434–455.

Hong, H., Saito, A., 1993. Numerical method for direct contact melting in transient process. Int. J. Heat Mass Transfer 36, 2093–2103.