A behavior analysis of multi-phase moving boundary using the CIP method
CIP ¹ýÀ» ÀÌ¿ëÇÑ
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- 2 -
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CIP (Cubic–Interpolated Propagation) is a method for propagating the solution of a system of equations across time steps. CIP is useful in solving hyperbolic partial differential equations. CIP is often compared to its cousin, CUP (Combined Unified Procedure).
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Nomenclature

$C_s$  The speed of sound waves
$e$  The specific internal energy
$g$  Gravitational acceleration
$t$  Time
$p$  Pressure
$u$  Velocity of $x$ direction
$v$  Velocity of $y$ direction
$\gamma$  The specific heat ratio
$\rho$  Density
$\nu$  Kinematic viscosity

Superscript

$n$  Nth time step

*  Contemporary value between n time step and n+1 time step

$n+1$  N+1th time step
1. ¼­·Ð

ÃÖ±ÙÄÄÇ»ÅÍÀÇ±Þ°ÝÇÑ¹ß´Þ°ú»õ·Î¿î¼öÄ¡ÇØ¼®±â¹ýÀÇ°³¹ß¿¡µû¶ó¿­À¯Ã¼°øÇÐÀÀ¿ëºÐ¾ß¿¡¼­Áö±Ý±îÁö´Â½ÇÇè¿¡ÀÇÁ¸ÇØ¿Ô´øÇö»óµé¿¡´ëÇÑ¼öÄ¡ÇØ¼®ÀÌȰ¹ßÇϰÔÁøÇàµÇ°íÀÒÀµÇ°í±×Áß´Ù»ó¿­À¯µ¿¹×»óº¯È­Çö»óÀºÇϳªÀÇ¹°¸®¿µ¿ª³»µ¥,±Ý¼ÓµîÀÇ°íü¿¡·¹ÀÌÀú¸¦Á¶»çÇÏ¿©³ìÀ̰íÁõ¹ß½ÃÄѼ­±¸¸ÛÀ»¶Õ¾î°¡´ÂÀӷ¸çÀϾ¸ç´õ¿í°¡¿­ÇϸéÁõ¹ßÀÌÀϾ´Ù.`

¹ßÇϰÔÁøÇàµÇ°íÀÒÀµÇ°í±×Áß´Ù»ó¿­À¯µ¿¹×»óº¯È­Çö»óÀºº¼¼öÀÖ´ÂÇÑ¿¹·Î·¹ÀÌÀú¸¦ÀÖ´ÂÇÑÀÖ À¸³ª¾ÆÁ÷µµÇö»ó¿¡´ëÇÑÀÌ·ÐÀÌ¸íÈ®È÷È®¸³µÇ¾îÀÖÁö¾Ê´Ù.`

1000 ¼­·±Çö»óÀ»¼öÄ¡ÀûÀ¸·ÎÇØ¼®ÇϱâÀ§ÇÑ½Ãµµ·ÎÁö±Ý±îÁö¿©·¯°¡ÁöÀÇ¼öÄ¡±â¹ýµéÀÌÀ¦½ÃµÇ¾î¿Ô´Ù.`

100 ¼­·±Çö»óÀ»¼öÄ¡ÀûÀ¸·ÎÇØ¼®ÇϱâÀ§ÇÑ½Ãµµ·ÎÁö±Ý±îÁö¿©·¯°¡ÁöÀÇ¼öÄ¡±â
¹ýÀº Harlow Amsden[1] ICE(Implicit Continuous Eulerian) SIMPLE, VOF(Volume of Fraction)[2]

³ª 3 ΛºøÀ¸·ÎÈ®ÀåÅ·ÀÌÈûµé°í±âÇÏÇÐÀûÀÎÇü»óÀÌº¹ÀâÇÒ°æ¿ì¿¡´ÂºÎÁ¤È®ÇÑ°á°ú°¡, °ÝÀÚ°¡¹°Áú°úÇÔ²²À̵¿ÇÏ´Â LAGRANGIAN

ALE(Arbitrary Lagrangian Eulerian)
CIP (Cubic–Interpolated Propagation) was introduced in 1985 by Yabe et al. [3-8]. CIP is a technique for computing the flow field by interpolating the data at the cell faces. It is a popular method due to its simplicity and accuracy.

CIP was developed as a method to improve the accuracy of the SIMPLE algorithm [7,8]. The SIMPLE algorithm is widely used in CFD simulations, and CIP helps in reducing the numerical diffusion and improving the accuracy of the solution. C-CUP (Cubic–Cubic–Upwind) is another method that is used in conjunction with CIP to further improve the accuracy of the simulation.
2. CIP \textit{\&} C-CUP \textit{\&}

2.1 CIP \textit{\&} C-CUP

CIP \textit{\&} C-CUP \textit{\&} 1 \textit{\&} \textit{\&} \textit{\&} \textit{\&}. \textit{\&} CIP \textit{\&} C-CUP \textit{\&} 1 \textit{\&} \textit{\&} \textit{\&} C-CUP

\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0 \tag{1}
\]

\[
f(x,t) = f(x - ut,0) \tag{2}
\]

Fig. 1 \textit{\&} 1(a) \textit{\&} 1(b) \textit{\&} 1(c) \textit{\&} \textit{\&} 1(d) \textit{\&} 1(e) \textit{\&} 1(f) \textit{\&} 1(g) \textit{\&} 1(h) \textit{\&} 1(i) \textit{\&} \textit{\&} 1(j) \textit{\&} \textit{\&} 1(k) \textit{\&} \textit{\&} 1(l) \textit{\&} \textit{\&} 1(m) \textit{\&} \textit{\&} 1(n) \textit{\&} \textit{\&} 1(o) \textit{\&} \textit{\&} 1(p) \textit{\&} \textit{\&} 1(q) \textit{\&} \textit{\&} 1(r) \textit{\&} \textit{\&} 1(s) \textit{\&} \textit{\&} 1(t) \textit{\&} \textit{\&} 1(u) \textit{\&} \textit{\&} 1(v) \textit{\&} \textit{\&} 1(w) \textit{\&} \textit{\&} 1(x) \textit{\&} \textit{\&} 1(y) \textit{\&} \textit{\&} 1(z) \textit{\&} \textit{\&} 1(\alpha) \textit{\&} \textit{\&} 1(\beta) \textit{\&} \textit{\&} 1(\gamma) \textit{\&} \textit{\&} 1(\delta) \textit{\&} \textit{\&} 1(\epsilon) \textit{\&} \textit{\&} 1(\zeta) \textit{\&} \textit{\&} 1(\theta) \textit{\&} \textit{\&} 1(\iota) \textit{\&} \textit{\&} 1(\kappa) \textit{\&} \textit{\&} 1(\lambda) \textit{\&} \textit{\&} 1(\mu) \textit{\&} \textit{\&} 1(\nu) \textit{\&} \textit{\&} 1(\xi) \textit{\&} \textit{\&} 1(\pi) \textit{\&} \textit{\&} 1(\rho) \textit{\&} \textit{\&} 1(\sigma) \textit{\&} \textit{\&} 1(\tau) \textit{\&} \textit{\&} 1(\upsilon) \textit{\&} \textit{\&} 1(\phi) \textit{\&} \textit{\&} 1(\chi) \textit{\&} \textit{\&} 1(\psi) \textit{\&} \textit{\&} 1(\omega) \textit{\&} \textit{\&} 1(\alpha) \textit{\&} \textit{\&} 1(\beta) \textit{\&} \textit{\&} 1(\gamma) \textit{\&} \textit{\&} 1(\delta) \textit{\&} \textit{\&} 1(\epsilon) \textit{\&} \textit{\&} 1(\zeta) \textit{\&} \textit{\&} 1(\theta) \textit{\&} \textit{\&} 1(\iota) \textit{\&} \textit{\&} 1(\kappa) \textit{\&} \textit{\&} 1(\lambda) \textit{\&} \textit{\&} 1(\mu) \textit{\&} \textit{\&} 1(\nu) \textit{\&} \textit{\&} 1(\xi) \textit{\&} \textit{\&} 1(\pi) \textit{\&} \textit{\&} 1(\rho) \textit{\&} \textit{\&} 1(\sigma) \textit{\&} \textit{\&} 1(\tau) \textit{\&} \textit{\&} 1(\upsilon) \textit{\&} \textit{\&} 1(\phi) \textit{\&} \textit{\&} 1(\chi) \textit{\&} \textit{\&} 1(\psi) \textit{\&} \textit{\&} 1(\omega) \textit{\&} \textit{\&} 1(\alpha) \textit{\&} \textit{\&} 1(\beta) \textit{\&} \textit{\&} 1(\gamma) \textit{\&} \textit{\&} 1(\delta) \textit{\&} \textit{\&} 1(\epsilon) \textit{\&} \textit{\&} 1(\zeta) \textit{\&} \textit{\&} 1(\theta) \textit{\&} \textit{\&} 1(\iota) \textit{\&} \textit{\&} 1(\kappa) \textit{\&} \textit{\&} 1(\lambda) \textit{\&} \textit{\&} 1(\mu) \textit{\&} \textit{\&} 1(\nu) \textit{\&} \textit{\&} 1(\xi) \textit{\&} \textit{\&} 1(\pi) \textit{\&} \textit{\&} 1(\rho) \textit{\&} \textit{\&} 1(\sigma) \textit{\&} \textit{\&} 1(\tau) \textit{\&} \textit{\&} 1(\upsilon) \textit{\&} \textit{\&} 1(\phi) \textit{\&} \textit{\&} 1(\chi) \textit{\&} \textit{\&} 1(\psi) \textit{\&} \textit{\&} 1(\omega)
Fig. 1(d) CIP 3

\[
\frac{x_i - x}{x_i - x_{i+1}} = \frac{F(x) - f_i}{f_{i+1} - f_i}
\]
\[ F(x) = \frac{x - x_i}{\Delta x} \left( f_{i+1} - f_i \right) + f_i^n \]  \hspace{1cm} (5)

\[ \Delta x = x_{i+1} - x_i \]  \hspace{1cm} (5)

\[ f_{i+1} = F(x_i - u \Delta t) = -\frac{u \Delta t}{\Delta x} \left( f_{i+1} - f_i \right) + f_i^n \]  \hspace{1cm} (6)

\[ x - x_i = u \Delta t \]  \hspace{1cm} (6)

\[ F(x) = ax^2 + bx + c \]  \hspace{1cm} (7)

\[ F(x_{i+1}) = f_{i+1}^n \]  \hspace{1cm} (8)

\[ F(x_i) = f_i^n \]  \hspace{1cm} (8)

\[ F(x_{i-1}) = f_{i-1}^n \]  \hspace{1cm} (8)

\[ F(x) = a(x - x_{i-1})^2 + b(x - x_{i-1}) + f_{i-1}^n \]  \hspace{1cm} (9)

\[ F(x_i) = f_i^n, \quad F(x_{i+1}) = f_{i+1}^n \]  \hspace{1cm} (9)
\[ a = \frac{f^n_{i+1} - 2f^n_i + f^n_{i-1}}{2\Delta x^2} \]  
\[ b = \frac{f^n_i - f^n_{i+1}}{\Delta x} - a\Delta x \]  
\[ \Delta x = x_i - x_{i-1} \]  

\[ f^{n+1}_i = f^n_i - \frac{u\Delta t}{2\Delta x} (f^n_{i+1} - f^n_{i-1}) + \frac{1}{2} \left( \frac{u\Delta t}{\Delta x} \right)^2 (f^n_{i+1} - 2f^n_i + f^n_{i-1}) \]  

\[ F_i(x) = a_iX^3 + b_iX^2 + \dot{f}_iX + f_i, \quad X = x - x_i \]  

\[ F_i(x_{i+1}) = F_i(x_{i+1}) \]  
\[ \frac{dF_i(x_{i+1})}{dx} = \frac{dF_i(x_{i+1})}{dx} \]  
\[ \frac{d^2F_i(x_{i+1})}{d^2x} = \frac{d^2F_i(x_{i+1})}{d^2x} \]  

\[ a_i\Delta x^3 + b_i\Delta x^2 + \dot{f}_i\Delta x + f_i = f_{i+1} \]
\[ 3a_i \Delta x^2 + 2b_i \Delta x + \dot{f}_i = \dot{f}_{i+1} \] (18)

\[ \Delta x = x_i^{i+1} - x_i \] .

(17) \[ a_i = \frac{(\dot{f}_i + \dot{f}_{i-1})}{\Delta x^2} + \frac{2(f_i - f_{i-1})}{\Delta x^3} \] (19)

\[ b_i = \frac{3(f_{i+1} - f_i)}{\Delta x^2} - \frac{(2\dot{f}_i + \dot{f}_{i+1})}{\Delta x} \] (20)

\[ \dot{f}_i = 2 \] 2 \[ \dot{f}_i = 3 \] 3

\[ f_i^{n+1} = F_i(x_i - u \Delta t) \] (22)

\[ \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0 \] (21)

\[ \frac{\partial}{\partial t} \left( \frac{\partial f}{\partial x} \right) + u \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) + \frac{\partial f}{\partial x} \frac{\partial u}{\partial x} = 0 \] (23)

\[ \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0 \] (24)
\[
\hat{f}_{i}^{n+1} = \frac{dF_i(x_i - u\Delta t)}{dx}
\] (25)

- \(i = 1, 2, 3\) -

- \(a_i, b_i\) -

\[
f_i^{n+1} = a_i \xi^3 + b_i \xi^2 + \hat{f}_i \xi + f_i
\] (26)

\[
\hat{f}_i^{n+1} = 3a_i \xi^2 + 2b_i \xi + \hat{f}_i, \quad \xi = -u\Delta t
\] (27)

CIP 2, 3 CIP CIP 1000 500 Fig. 4 Fig. 5 tangent Fig. 6 Fig. 7
Fig. 8

Fig. 6

CIP tangent (phase)

CIP overshoot

\[ h = \tan \left[ \pi \left( f - \frac{1}{2} \right) \right] \]

\[ f = \frac{\arctan(h)}{\pi} + \frac{1}{2} \] (28)

\( f = 0, 1 \)
2.2 ¹æÁ¤½ÄÀÇ

2.1 CIP: ¹æÁ¤½ÄÀÇ Àý ¿ø¸®¿¡¼­ ¿³¸í ÇÑ®ÔÀº ½Ö°î¼± ¹æÁ¤½ÄÀÇ.

\[
\frac{\partial f}{\partial t} + \frac{\partial fu}{\partial x} = g
\]  
(29)

\[ (29) \] CIP: Àý ÀÎ ÇØ¹ýÀ̾ú°í ÀÌ Àý¿¡¼­´Â ²±Çü(1)À­À­ Àý ÀÎ ÇØ¹ýÀ̾ú°í ÀÌ Àý¿¡¼­´Â

\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = G
\]  
(30)

\[
G = g - f \frac{\partial u}{\partial x}
\]  
(30) CIP: Àý¿¡¼­´Â \( \dot{f} \) (30) Àý¿¡¼­´Â \( \dot{f} \) àÈÁ¤¸® Àº ÇØ¹ýÀ̾ú°í ÀÌ Àý¿¡¼­´Â

\[
\frac{\partial \dot{f}}{\partial t} + u \frac{\partial \dot{f}}{\partial x} = G - f \frac{\partial u}{\partial x}
\]  
(31)

CIP (30) (31) 2 ÇØ¹ýÀ̾ú°í Àº ÇØ¹ýÀ̾ú°í Àº

\[
\frac{\partial f}{\partial t} = G
\]  
(32)

\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0
\]  
(33)

\[
\frac{\partial \dot{f}}{\partial t} = G - f \frac{\partial u}{\partial x}
\]  
(34)
\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0 \tag{35}
\]

\[
\text{(32)} \quad \text{(34)} \quad \text{(33)} \quad \text{(35)}
\]

\[f^* = f^n + G_i \Delta t \tag{36}\]

\[\dot{G}_i = \frac{G_{i+1} - G_{i-1}}{2\Delta x} = \frac{f^n_{i+1} - f^n_{i-1} + f^n_{i+1} + f^n_{i-1}}{2\Delta x \Delta t} \tag{37}\]

\[\dot{f}^* = \dot{f}^n + \frac{f^*_{i+1} - f^*_{i-1} + f^*_{i+1} + f^*_{i-1}}{2\Delta x} - \dot{f}^n \frac{u^n_{i+1} - u^n_{i-1}}{2\Delta x} \Delta t \tag{38}\]

\[G_{i+1} \quad G_{i-1} \]

B. 

\[f^* \quad \dot{f}^* \quad f^{n+1} \quad \dot{f}^{n+1} \]
（29）の式を用いればよい。
2.3 C-CUP

2.3.1 C-CUP

C-CUP (CIP-Combined Unified Procedure). C-CUP

\[
\frac{\partial \hat{f}}{\partial t} + (\bar{u} \cdot \nabla) \hat{f} = \hat{G}
\]  
(39)

\[
\hat{f} = (\rho, \bar{u}, p), \quad \hat{G} = (-\rho \nabla \cdot \bar{u}, -\frac{\nabla p}{\rho}, -\gamma p \nabla \cdot \bar{u})
\]

(39) (non-convection stage)

\[
\frac{\partial \hat{f}}{\partial t} = \hat{G}
\]  
(40)

(convection stage)

\[
\frac{\partial \hat{f}}{\partial t} + (\bar{u} \cdot \nabla) \hat{f} = 0
\]  
(41)

\[
\frac{\rho^* - \rho^n}{\Delta t} = -\rho^n \nabla \cdot \bar{u}^n
\]  
(42)

\[
\frac{\bar{u}^* - \bar{u}^n}{\Delta t} = -\frac{\nabla p^*}{\rho^*}
\]  
(43)
\frac{\ddot{u} - \dddot{u}}{\Delta t} = \ddot{Q}_u \quad (44)

\frac{p^{**} - p^*}{\Delta t} = -\gamma p^* \nabla \cdot \ddot{u} \quad (45)

\frac{p^* - p^{**}}{\Delta t} = \ddot{Q}_p \quad (46)

\n
\frac{\nabla \cdot \dddot{u} - \nabla \cdot \dddot{u}^*}{\Delta t} = -\frac{\nabla^2 p^{**}}{\rho^*} \quad (47)

\n
\frac{\nabla^2 p^{**}}{\rho^*} = \frac{p^{**} - p^*}{\gamma \rho^* \Delta t^2} + \frac{\nabla \cdot \dddot{u}^*}{\Delta t} \quad (48)
\[ (48) \quad \text{MAC (marker and cell)} \]

\[ (48) \quad \text{MAC} \quad \text{MAC} \]

\[ \begin{align*}
&\text{MAC} \quad \text{MAC} \\
&\text{MAC} \quad \text{MAC} \quad \text{MAC} \quad \text{MAC} \quad \text{MAC} \\
&\text{MAC} \quad \text{MAC} \\
&\text{MAC} \quad \text{MAC} \quad \text{MAC} \quad \text{MAC} \\
\end{align*} \]

\[ \frac{p^{n+1} - p^n}{\gamma p^n \Delta t} = 0 \quad \text{(48)} \]

\[ C_s = \sqrt{\frac{\gamma p}{\rho}} \quad \text{(49)} \]

\[ \nabla^2 p^{n+1} = \nabla \cdot \vec{u}^n \quad \text{(50)} \]

\[ \text{Poisson} \quad \text{Poisson} \quad \text{Poisson} \quad \text{Poisson} \quad \text{Poisson} \quad \text{Poisson} \]

\[ \begin{align*}
&\nabla^2 p^{n+1} = \nabla \cdot \vec{u}^n \\
&\frac{\nabla^2 p^{n+1}}{\rho^n} = \frac{\nabla^2 p^{n+1}}{\rho^n} \\
&\frac{\nabla^2 p^{n+1}}{\rho^n} = 0 \\
&\text{(48)} \\
\end{align*} \]

\[ \begin{align*}
&\frac{\nabla^2 p^{n+1}}{\rho^n} = \frac{\nabla^2 p^{n+1}}{\rho^n} \\
&\text{(48)} \\
\end{align*} \]

\[ \begin{align*}
&\frac{\nabla^2 p^{n+1}}{\rho^n} = \frac{\nabla^2 p^{n+1}}{\rho^n} \\
&\frac{\nabla^2 p^{n+1}}{\rho^n} = \gamma p^n \nabla \cdot \vec{u}^n \\
&\text{(51)} \\
\end{align*} \]
Poison equation: 

\[ \frac{\bar{u}^* - \bar{u}^n}{\Delta t} = -\frac{\nabla p^{**}}{\rho^n} \]  

(52)

\[ \rho^* - \rho^n = \frac{\rho^n}{\gamma p^n} (p^{**} - p^n) \]  

(53)

\[ (46) \quad p^* \quad u^* \quad \rho^* \quad \text{in CIP} \quad p^{n+1}, u^{n+1}, \rho^{n+1} \]
2.3.2 C-CUP

\[
\frac{\partial \mathbf{f}}{\partial t} + u \frac{\partial \mathbf{f}}{\partial x} = \mathbf{G} \tag{54}
\]

\[
\mathbf{f} = (\rho, u, e), \quad \mathbf{G} = \left(-\rho \frac{\partial u}{\partial x}, -\frac{1}{\rho} \frac{\partial p}{\partial x}, -\frac{p}{\rho} \frac{\partial u}{\partial x}\right)
\]

\(\mathbf{f}\) (staggered grid).

\[
\frac{\rho_i^n - \rho_i^*}{\Delta t} = -\rho_i^n \frac{u_{i+1/2}^n - u_{i-1/2}^n}{\Delta x} \tag{55}
\]

\[
\frac{u_{i+1/2}^* - u_{i+1/2}^n}{\Delta t} = -\frac{1}{\rho_{i+1}^n} \frac{p_{i+1}^n - p_i^n}{\Delta x} \tag{56}
\]

\[
\frac{e_i^* - e_i^n}{\Delta t} = -\frac{p_i^n}{\rho_i^n} \frac{u_{i+1/2}^* - u_{i-1/2}^* + u_{i+1/2}^n - u_{i-1/2}^n}{2\Delta x} \tag{57}
\]
\[ u_{av} = \left( \frac{u_{i+1/2} + u_{i-1/2}}{2} \right) \]

\[ q_i = \begin{cases} a \left( -\rho_i C_i \Delta u + \frac{\gamma + 1}{2} \rho_i \Delta u^2 \right) & \text{if } \Delta u < 0 \\ 0 & \text{if } \Delta u \geq 0 \end{cases} \]  \hspace{1cm} (58)

\[ \Delta u = u_{i+1/2} - u_{i-1/2} \]
2.3.3 C-CUP

\[ \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} = g \]  \hspace{1cm} (59)

\( \Delta x \) \quad \Delta y \quad \Delta z

\begin{align*}
\Delta z & = (i,j) - (i,j+1) - (i+1,j+1) - (i+1,j) \quad 3 \quad 2 \quad 1 \\
\end{align*}

\[ F_{ij}(x,y) = \left[ (A_{1,ij}X + A_{2,ij}Y + A_{3,ij})X + A_{4,ij}Y + \partial_x f_{ij} \right]X \\
+ \left[ (A_{5,ij}Y + A_{6,ij}X + A_{7,ij})Y + \partial_y f_{ij} \right]Y + f_{ij} \]  \hspace{1cm} (60)

\[ f_{ij} = f_{i,j}^0 + g_{i,j} \Delta t \]  \hspace{1cm} (61)

\[ \partial_x f_{ij} = \partial_x f_{ij}^0 - \frac{f_{i,j}^{n+1} - f_{i,j}^{n} - f_{i+1,j}^{n} + f_{i+1,j}^{n-1}}{2 \Delta x} \\
- \frac{\left( u_{i,j} - u_{i,j+1} \right) \Delta t}{2 \Delta x} - \frac{\left( v_{i+1,j} - v_{i,j+1} \right) \Delta t}{2 \Delta x} \]  \hspace{1cm} (62)

\[ \partial_y f_{ij} = \partial_y f_{ij}^0 - \frac{f_{ij+1}^{n+1} - f_{ij+1}^{n} - f_{ij+1}^{n} + f_{ij+1}^{n-1}}{2 \Delta y} \\
- \frac{\left( u_{i+1,j} - u_{i,j+1} \right) \Delta t}{2 \Delta y} - \frac{\left( v_{i,j+1} - v_{i+1,j} \right) \Delta t}{2 \Delta y} \]  \hspace{1cm} (63)

\[ f_{ij}^{n+1} = F_{ij} \left( x_i - u_i \Delta t, y_j - v_i \Delta t \right) \partial_x f_{ij}^{n+1} = \partial_x F_{ij}, \partial_y f_{ij}^{n+1} = \partial_y F_{ij} \]
\[ f_{x,j}^{n+1} = \left[ (A1_{i,j} \xi + A2_{i,j} \eta + A3_{i,j} \xi) + A4_{i,j} \eta + \partial_x f_{i,j}^n \right] + \left[ (A5_{i,j} \eta + A6_{i,j} \xi + A7_{i,j} \eta) \right] + \partial_x f_{i,j}^n \]  

(64)

\[ \partial_x f_{x,j}^{n+1} = (3A1_{i,j} \xi + 2A2_{i,j} \eta + 2A3_{i,j} \xi) + \left( A4_{i,j} + A6_{i,j} \eta \right) \eta + \partial_x f_{i,j}^n \]  

(65)

\[ \partial_y f_{x,j}^{n+1} = (3A5_{i,j} \eta + 2A6_{i,j} \xi + 2A7_{i,j} \eta) \eta + \left( A4_{i,j} + A2_{i,j} \xi \right) \xi + \partial_y f_{i,j}^n \]  

(66)

\[
\begin{bmatrix}
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} = g
\end{bmatrix}
\]

(67)

\[
\begin{bmatrix}
\frac{\partial p}{\partial t} + \frac{\partial p u}{\partial x} + \frac{\partial p v}{\partial y} = 0
\end{bmatrix}
\]

(68)

\[
\begin{bmatrix}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x}
\end{bmatrix}
\]

(69)

\[
\begin{bmatrix}
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y}
\end{bmatrix}
\]

(70)

\[
\begin{bmatrix}
\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} = - \frac{p}{\rho} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)
\end{bmatrix}
\]

(71)
\[ \rho^*_{i,j} - \rho^n_{i,j} = -\rho^n_{i,j} \left( \frac{u^n_{i+1/2,j} - u^n_{i-1/2,j}}{\Delta x} + \frac{v^n_{i,j+1/2} - v^n_{i,j-1/2}}{\Delta y} \right) \] (72)

\[ \frac{u^n_{i+1/2,j} - u^n_{i+1/2,j}}{\Delta t} = -\frac{2}{\rho^*_{i+1,j} + \rho^*_{i,j}} \left( \frac{p^n_{i+1,j} - p^n_{i,j}}{\Delta x} \right) \] (73)

\[ \frac{v^n_{i,j+1/2} - v^n_{i,j+1/2}}{\Delta t} = -\frac{2}{\rho^*_j + \rho^*_i} \left( \frac{p^n_{i,j+1} - p^n_{i,j}}{\Delta y} \right) \] (74)

\[ \frac{e^*_{i,j} - e^n_{i,j}}{\Delta t} = -\frac{p^n_{i,j}}{\rho^n_{i,j}} \left( \frac{\text{DIV}^n + \text{DIV}^*}{2} \right) \] (75)

**C-CUP**

\[ \text{DIV} = \left( \frac{u^n_{i+1/2,j} - u^n_{i-1/2,j}}{\Delta x} + \frac{v^n_{i,j+1/2} - v^n_{i,j-1/2}}{\Delta y} \right) \]
\[ u_{1,i,j} = \left( \frac{u_{i+1/2,j}^n + u_{i-1/2,j}^n}{2} \right) \quad \nu_{1,i,j} = \left( \frac{v_{i+1/2,j}^n + v_{i-1/2,j}^n}{2} \right) \]  

(76)

\[ u = u_{i,j}^n, v = \left( \frac{v_{i+1/2,j+1/2}^n + v_{i-1/2,j-1/2}^n + v_{i+1/2,j-1/2}^n + v_{i-1/2,j+1/2}^n}{4} \right) \]  

(77)

\[ u = \left( \frac{u_{i+1/2,j+1/2}^n + u_{i-1/2,j-1/2}^n + u_{i+1/2,j-1/2}^n + u_{i-1/2,j+1/2}^n}{4} \right), v = v_{i,j}^n \]  

(78)
Fig. 1 Modelling of interpolation
Fig. 2 A linear interpolation
Fig. 3 A spline interpolation
Fig. 4 The profile after 1000 timesteps for each algorithm as initial profile is square and triangle

(a) upwind method  (b) Lax-Wendroff method

(c) CIP method  (d) CIP method using tangent method
Fig. 5 Profile after 500 time step as initial profile is square
and profile at same condition with tangent method
Fig. 6 Profile at each 100 time step when initial profile is square and moving velocity is $u > 0$ and $v > 0$
Fig. 7 Profile at each 100 time step when initial profile is square and moving velocity is $u > 0$ and $v < 0$
Fig. 8 Profile at each 100 time step when initial profile is circle and moving velocity is \( u > 0 \) and \( v > 0 \)
3. 二次元流動の観察

3.1 二次元流動の観察

Fig. 9 に示すように、速度分布を測定した結果を示す。速度は流れ方向に経時的に変化し、流体のレイノルズ数が210の時に最大値を示し、その後減衰する。グラフの色は速度分布を示しており、0.005m/s のレートでレイノルズ数を測定した。レイノルズ数は22×22の領域で計算され、62×62の領域で計算される。

図 2 に示すように、流れの測定点は、x, y の個々の値を示している。流れの速度成分 (u, v) は0で、(v=0) の場合 u の平均速度を計算する。

3.2 エキゾン

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = -\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{79}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{80}
\]
\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{81}
\]

\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = -\gamma p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{82}
\]

3.3 Equation

\[
\text{(79)} \quad \text{Consider the equations (79).}
\]

\[
\text{(79)} \quad \text{The equations are solved using the C-CUP method.}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = 0 \tag{83}
\]

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{84}
\]

C-CUP \quad \text{The Poisson equation is solved using the C-CUP method.}

\[
\nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad \text{The Poisson equation is solved using the C-CUP method.}
\]
\[
\frac{\ddot{u}^* - \ddot{u}^n}{\Delta t} = -\frac{\nabla p^*}{\rho^n} \tag{85}
\]

\[
\frac{\nabla \cdot \ddot{u}^* - \nabla \cdot \ddot{u}^n}{\Delta t} = -\frac{\nabla^2 p^*}{\rho^n} \tag{86}
\]

\[
\nabla \cdot \ddot{u}^* = \frac{\nabla^2 p^*}{\rho^n} \Delta t + \nabla \cdot \ddot{u}^n \tag{87}
\]

\[
\frac{p^* - p^n}{\Delta t} = -\gamma p^n \nabla \cdot \ddot{u}^* \tag{88}
\]

\[
\frac{\nabla^2 p^*}{\rho^n} = \frac{\nabla \cdot \ddot{u}^*}{\Delta t} + \frac{p^* - p^n}{\gamma p^n \Delta t^2} \tag{89}
\]

\[
\gamma p^n = C_s^2 \rho \tag{89}
\]

\[
\nabla^2 p^* = \frac{\nabla \cdot \ddot{u}^*}{\Delta t} \tag{90}
\]

\[
\text{Poisson} \tag{89}
\]
\[ c \frac{\Delta t}{\Delta x} \leq 1 \]  \hspace{1cm} (91)

\[ c \leq 0.0167 \]
\[
\frac{u_{ij}^* - u_{ij}^n}{\Delta t} = \frac{1}{\rho} \frac{p_{ij}^* - p_{i-1,j}^n}{\Delta x} + \sqrt{\left(\frac{u_{i+1,j}^n - 2u_{ij,j}^n + u_{i-1,j}^n}{\Delta x^2} + \frac{u_{i,j+1}^n - 2u_{ij,j}^n + u_{i,j-1}^n}{\Delta y^2}\right)}
\]

(92)

\[
\rho \cdot u_{ij}^*, v_{ij}^*, p_{ij}^* \square \text{ CIP} \quad \square \text{ CIP} \quad \square \text{ CIP}
\]

- 45 -
\[ u = u_{i,j}, v = \left( \frac{v_{i+1,j+1/2}^n + v_{i+1,j-1/2}^n + v_{i,j+1/2}^n + v_{i,j-1/2}^n}{4} \right) \] (93)

\[ u = \left( \frac{u_{i+1/2,j+1/2}^n + u_{i-1/2,j+1/2}^n + u_{i+1/2,j-1/2}^n + u_{i-1/2,j-1/2}^n}{4} \right) v = v_{i,j} \] (94)

\[ u_{i,j} = \left( \frac{u_{i+1/2,j}^n + u_{i-1/2,j}^n}{2} \right) v_{i,j} = \left( \frac{v_{i,j+1/2}^n + v_{i,j-1/2}^n}{2} \right) \] (95)

\[ f_{i,j}^{n+1} = 2.3.3 \quad \text{2.3.3} \]

\[ f_{i,j}^{n+1} = \left[ \left( A_{1,i,j}^n \xi + A_{2,i,j}^n \eta + A_{3,i,j}^n \xi + A_{4,i,j}^n \eta + \partial_z f_{i,j}^n \right) \right. \]
\[ + \left[ \left( A_{5,i,j}^n \eta + A_{6,i,j}^n \xi + A_{7,i,j}^n \eta + \partial_z f_{i,j}^n \right) \right] \] (96)
3.4 3.4

... (vortex) ... C-CUP ... SIMPLE ...

Fig. 10

Re=210 22x22  Fig. 11

CIP ... SIMPLE ...

Fig. 12  Re=210 22x22, Fig. 14  Re=500 32x32

Fig. 13  Re=500 32x32, Fig. 15  Re=210 42x42

Fig. 16  Re=210 42x42  Fig. 17

Re=210 62x62 ...

CIP ...

Fig. 18  (y = 3.9x10^-2 m, y = 2.4x10^-2 m)
Fig. 18에 보이는 것으로, 15%의 유 효성은 보인습니다.

SIMPLE의 경우, 흔히 밑줄을 사용하여 이러한 정보를 나타냅니다.
Fig. 9 Schematic diagram with boundary conditions

- 49 -
Fig. 10 Grid
Fig. 11 Distribution of velocity vectors in cavity at Re=210 by 22×22
Fig. 12 Distribution of streamline in cavity at Re=210 by 22×22
Fig. 13 Distribution of velocity vectors in cavity at Re=500 by 32x32
Fig. 14 Distribution of streamline in cavity at Re=500 by 32×32
Fig. 16 Distribution of streamline in cavity at Re=210 by 42×42
Fig. 17 Distribution of streamline in cavity at Re=210 by 62×62
Fig. 18 Comparison of $u$ with CIP method and SIMPLE at $y=3.9\text{cm}$ and $2.4\text{cm}$
4. ÇØ¼®¸ðµ¨°ú

4.1 ÇØ¼®¸ðµ¨°ú

VOF (Volume of fluid)™ MAC ¹ý°ú´Â °æ°èÁ¶°Ç ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À¨Á÷¼±º¸°£ÇÏ¿© ½Ã. 3 ¹æ¹ýÀ» ¾²°í ÀÖÀ¸¸ç über ¼÷ Â±º¸°£ÇÏ¿© ¼öÁ÷ÀÀ·ÂÁ¶°ÇÀ» »ç¿ëÇØ 

MAC ¹ýÀº ¼öÄ¡ÇØ¹ý¿¡¼­´Âº¹ÀâÇÑ °æ°èÁ¶°ÇÀ» ¼öÄ¡ÇØ¹ý¿¡¼­˚å, ¼öÄ¡ÇØ¹ý¿¡¼­ÀÇ ¾Ð·ÂÀ» Á÷¼±º¸°£ÇÏ¿© »ç¿ëÇØ ÁÖ¾îÁö¹Ç·Î ¹æ´ëÇØÁö¸ç, ¾è»ê¿µ¿ª ÁÖÀ§ÀÇ °è»êµÈ ÀÎÁ¢ÇÑ °è»ê¼¿ (cell) ¹æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ǷÎ ±â °æ°è°¡ ¹ºÎ¿©ÇÏ¿© ÀÚÀ¯Ç¥¸éÀÇ À§Ä¡¸¦ ¾Æ´Â ¹ºÀº À§Ä¡¸¦ ¾î·Á¿î ÀÏÀ̹ราชการificant
C-CUP bench mark problem C-CUP bench mark problem.

Fig. 19 bench mark problem C-CUP bench mark problem.

Fig. 19 bench mark problem C-CUP bench mark problem.

bench mark problem C-CUP bench mark problem.

bench mark problem C-CUP bench mark problem.

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bench mark problem C-CUP bench mark problem.

bench mark problem C-CUP bench mark problem.
\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = -\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{97}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - g \tag{98}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \tag{99}
\]

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = -\gamma \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{100}
\]

4.3 企♂Р□□

4.2 企♂Р□□

\[ q_i = \alpha \left( -\rho, C, \Delta u + \frac{\gamma + 1}{2} \rho, \Delta u^2 \right) \text{ if } \Delta u < 0 \]

\[ = 0 \quad \text{if } \Delta u \geq 0 \tag{101} \]
\[ \frac{\partial u}{\partial t} = - \frac{1}{\rho} \frac{\partial p}{\partial x} - g \]  \hfill (102)

\[ \frac{\bar{u}^n - u^n}{\Delta t} = - \nabla p^* - g \]  \hfill (103)

Poisson \hfill (103)

\[ \frac{\nabla \cdot \bar{u}^n - \nabla \cdot \bar{u}^n}{\Delta t} = - \frac{\nabla^2 p^*}{\rho^n} \]  \hfill (104)

\[ \frac{p^* - p^n}{\Delta t} = - \gamma p^n \nabla \cdot \bar{u}^* \]  \hfill (105)
\[ \frac{\nabla^2 p^*}{\rho^n} = \frac{\nabla \cdot \bar{u}^n + p^* - p^n}{\Delta t} + \frac{\gamma p^n \Delta t^2}{\rho^n} \] (106)

MAC \quad (106) \quad C-\quad CUP \quad (106) \quad SOR(Successive OverRelaxation) \quad 0.2 \quad (106)

\[ \frac{u^{**} - u^n}{\Delta t} = -\frac{1}{\rho} \frac{p_{i,j}^* - p_{i+1,j}^n}{\Delta x} - g \] (107)

\[ \frac{u^* - u^{**}}{\Delta t} = \left( \frac{xvis_{i,j} - xvis_{i+1,j}}{\Delta x} \times \frac{2}{\rho_{i+1,j} + \rho_{i,j}} \right) \] (108)

\[ xvis(i,j) \quad (107) \quad \rho_{i,j}^* \]
\[ u_{i,j}, \quad v_{i,j} \quad \text{and} \quad p_{i,j} \quad \text{on} \quad \Omega. \]

\[ x \quad u \quad y \quad v \quad \text{on} \quad \Omega. \]

\[ \text{Using} \quad (93), \quad (94) \quad \text{and} \quad (95) \quad \text{we have} \quad \text{CIP} \quad \text{as follows} \quad \text{on} \quad \Omega. \]

\[ \partial_x p^* , \partial_y p^* , \partial_x u^* , \partial_y u^*, \partial_x v^* , \partial_y v^*, \partial_x e^* , \partial_y e^* \quad \text{on} \quad \Omega. \]

\[ \text{Using} \quad \text{CIP} \quad \text{for} \quad u^{n+1}_{i,j}, v^{n+1}_{i,j} \quad \text{and} \quad p^{n+1}_{i,j} \quad \text{we have} \quad \text{CIP} \quad \text{as follows} \quad \text{on} \quad \Omega. \]

\[ \rho^{n+1}_{i,j}, u^{n+1}_{i,j}, v^{n+1}_{i,j} , p^{n+1}_{i,j} \quad \rho^n_{i,j}, u^n_{i,j}, v^n_{i,j}, p^n_{i,j} \quad \text{on} \quad \Omega. \]

- 64 -
4.4 The Results

Fig. 22 shows the results of the MAC[9] and VOF[2] methods. The MAC method is able to capture the interface between the two fluids accurately, while the VOF method shows some numerical diffusion. The CIP method also provides good results, with minimal numerical diffusion compared to the other two methods.
air
initial condition
$u = 0, \ v = 0$

water
initial condition
$u = 0, \ v = 0$

Fig. 19 Schematic diagram with boundary conditions
Fig. 21 Fluid configuration of marker particles for the broken dam

at times $t=0, 0.5, 1.0, 1.5, 2.0, 2.5$. 
Fig. 22 Velocity vectors and fluid configuration for broken dam problem at times 0.0, 0.9, 1.4, 2.0
Fig. 23 Density distribution and fluid configuration for broken dam problem
5. นี่

... นี่ CIP นี่ CIP C-CUP นี่... นี่ CIP นี่ CIP C-CUP นี่... นี่ CIP นี่ CIP C-CUP นี่... SIMPLE นี่ SIMPLE นี่ CIP นี่ SIMPLE นี่ SIMPLE นี่... นี่ CIP นี่ SIMPLE นี่ SIMPLE นี่... นี่ CIP นี่ SIMPLE นี่ SIMPLE นี่... นี่ CIP นี่ SIMPLE นี่ SIMPLE นี่...

1. C-CUP ¹ýÀÇ 2 Â÷¿ø

C-CUP ¹ýÀ» ¼³¸íÇÑ 2 Â÷¿ø¿¡¼­ÀÇ ¼öÄ¡°è»ê ÇÁ·Î±×·¥¿¡¼­´Â ÀÎ °æ ¿ì¿¡¸¸ »ç¿ëÇÒ ÀÖ¾ú´Ù ±×·¯³ª ¹ü¿ë¼ºÀ» °¡Áø °¡Áö°í ÇÁ·Î±×·¥¿¡¼­´Â À̵¿¼Óµµ ÀÇ ¼ò ºÎÈ£¿¡ »ó°ü¾øÀÌ ÇϳªÀÇ ½ÄÀ» °¡Áö°í ÇϹǷÎ ±¸¼ºµÇµµ, ·Ï¸ÕÀú ·î °¡ÁöÀÇ º¯¼ö¸¦ »ç¿ëÇØ¾ß ÇÑ´Ù.

½ÇÁ¦ÀÇ ÇÁ·Î±×·¥Àº FORTRAN ¹ýÀ» ¹Ù·Î ¹ÚºÎºÐÀÌ ¹Ù·Î ÀÌ À½ÀÇ ¹ÚºÎºÐÀÌ´Ù ÀÌ À½ÀÇ ¹ÚºÎºÐÀÌ´Ù.

subroutine DCIP ¹ýÀ» CIP ¹ýÀ» ¹ìÁ·

    dx2=dx*dx
    dx3=dx2*dx
    dy2=dy*dy
    dy3=dy2*dy
    do 130 j=1,ny
    do 130 i=1,nx
      xx=-u(i,j)*dt
      yy=-v(i,j)*dt
      isn=sign(1.0,u(i,j)): 

sign  u(i,j):  0  1
u(i,j):  0  1.isn  0  1
\[ u(i,j) \] 0 \[ 0 \] 0 \[ 0 \] isn=1.0 \[ 0 \] 0 \[ 0 \] isn=-1.0

\[ jsn=\text{sign}(1.0,v(i,j)) \]

\[ im1=i-isn \]

\[ jm1=j-jsn \]

\[ a1=\frac{(gx(im1,j)+gx(i,j)) \cdot dx \cdot isn -2.0d0 \cdot (f(i,j)-f(im1,j))}{(dx^2 \cdot isn)} \]

\[ e1=\frac{3.0d0 \cdot (f(im1,j)-f(i,j)) + (gx(im1,j)+2.0d0 \cdot gx(i,j)) \cdot dx \cdot isn}{dx^2} \]

\[ b1=\frac{(gy(i,jm1)+gy(i,j)) \cdot dy \cdot jsn -2.0d0 \cdot (f(i,j)-f(i,jm1))}{(dy^2 \cdot jsn)} \]

\[ f1=\frac{3.0d0 \cdot (f(i,jm1)-f(i,j)) + (gy(i,jm1)+2.0d0 \cdot gy(i,j)) \cdot dy \cdot jsn}{dy^2} \]

\[ tmp=f(i,j)-f(i,jm1)-f(im1,j)+f(im1,jm1) \]

\[ tmq=gy(im1,j)-gy(i,j) \]

\[ dl=(-tmp-tmq \cdot dy \cdot jsn)/(dx \cdot dy^2 \cdot isn) \]

\[ cl=(-tmp-(gx(i,jm1)-gx(i,j)) \cdot dx \cdot isn)/(dx^2 \cdot dy \cdot jsn) \]

\[ gl=(-tmp+cl \cdot dx^2)/(dx \cdot isn) \]

\[ fn(i,j)=((a1 \cdot xx+cl \cdot yy+el) \cdot xx+g1 \cdot yy+gx(i,j)) \cdot xx+((bl \cdot yy+dl \cdot xx+fl) \cdot yy+g \text{y}(i,j)) \cdot yy+f(i,j) \]

\[ gxn(i,j)=(3.0d0 \cdot a1 \cdot xx+2.0d0 \cdot (cl \cdot yy+el)) \cdot xx+(dl \cdot yy+g1) \cdot yy+gx(i,j) \]

\[ gyn(i,j)=(3.0d0 \cdot b1 \cdot yy+2.0d0 \cdot (cl \cdot xx+fl)) \cdot yy+(cl \cdot xx+g1) \cdot xx+gy(i,j) \]
2. (broken dam)

4.2 (97), (98), (99), (100)

\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = - \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{97}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} - g \tag{98}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} \tag{99}
\]

\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = - \frac{\rho}{\rho} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{100}
\]

<table>
<thead>
<tr>
<th>( \bar{x} )</th>
<th>( \bar{h} )</th>
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</thead>
<tbody>
<tr>
<td>( \bar{h} = h )</td>
<td>( \bar{h} = \rho_0 gh = \rho_0 \bar{u}^2 )</td>
</tr>
<tr>
<td>( \bar{u} = \sqrt{gh} )</td>
<td>( \bar{t} = \frac{h}{\sqrt{gh}} )</td>
</tr>
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</table>

\[ u^* = \frac{u}{\bar{u}} \]
\[
\frac{\partial p^*}{\partial t} + u^* \frac{\partial p^*}{\partial x} + v^* \frac{\partial p^*}{\partial y} = -\rho \left( \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} \right) \]  
(97-1)

\[
\frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x} - 1 \]  
(98-1)

\[
\frac{\partial v^*}{\partial t} + u^* \frac{\partial v^*}{\partial x} + v^* \frac{\partial v^*}{\partial y} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y} \]  
(99-1)

\[
\frac{\partial p^*}{\partial t} + u^* \frac{\partial p^*}{\partial x} + v^* \frac{\partial p^*}{\partial y} = -\gamma p \left( \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} \right) \]  
(100-1)
Abstract

A behavior analysis of multi-phase moving boundary using the CIP method

Park Jun Hong
Dept. of Mechanical Engineering
The Graduate School
Kyung Hee Univ., KOREA

The CIP(Cubic-Interpolated Propagation) method which can solve together compressible and incompressible fluid is used to calculate multi-phase moving boundary. This method can treat solid, liquid and gas phases simultaneously and can trace a sharp interface even with one grid and provides a stable and less diffusive result even in a high-CFL computation.

As test problem, C-CUP(CIP Combined Unified Procedure) was applied to cavity flow and broken dam, which have been verified by lots of numerical techniques and experiments.

In the cavity flow, the distribution of velocity calculated by the CIP method shows a good agreement with that of SIMPLE.
In the simulation on collapsing process of dam, exact and sharp boundary was obtained on flowing water, although simple boundary conditions are used only on the rectangular region not water-air interface.